Measuring the Cavity-Pulling Coefficient of Active Optical Clock via Pump Laser Modulation

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***Summary*—The active optical clock (AOC) takes advantage of the narrowed linewidth and cavity pulling suppression of the bad cavity laser, making it a candidate for the next generation of optical clock. We propose a new experimental method to measure the cavity-pulling effect of a four-level AOC based on thermal cesium atoms. After modulating the frequency of the 459 nm pumping laser, atoms in different velocity groups are pumped to the $7P_{1/2}$ energy level. Therefore, population inversion is achieved in separate velocity groups, and sidebands are created in the gain profile. By measuring the central frequency of the beat spectrum between the zeroth order AOC output and the sidebands, the cavity-pulling coefficient can be calculated. This method has the advantage of not requiring any additional optical frequency reference, and can be adopted in AOC systems other than those using thermal atoms. Furthermore, our proposed method for cavity-pulling measurement can serve as a basis for the characterization of AOCs.**

Keywords—active optical clock, bad cavity laser, cavity pulling

I. Introduction

Optical clocks, with local oscillator frequencies 4 to 5 orders above that of microwave clocks, can achieve better frequency stability and accuracy than their microwave counterparts, reaching towards or even surpassing the 10^{-18} level [1-5]. Such high precision performance enables optical clocks to be used in applications such as geodesy [2,6], fundamental physics testing [7,8] and detection of gravitational waves [9]. With most high-precision optical clocks, in order to interrogate ultra-narrow linewidth transitions, the local oscillator lasers have to be pre-stabilized to high finesse optical cavities [10,11]. Despite efforts to reduce the thermal noise by cooling the optical cavity to cryogenic temperatures, the thermo-mechanical Brownian noise of the reference cavity remains a limiting factor to optical clock performance [12]. In light of this issue, in order to circumvent the problem of cavity thermal noise, the active optical clock (AOC) was proposed [13].

The AOC is based on the principle of bad-cavity laser, which has a gain linewidth narrower than the linewidth of the laser cavity [14]. They have the characteristic of a quantum-limited linewidth narrower than that given by the Schawlow-Townes formula, as well as suppression of cavity pulling [15]. These properties make the AOC a promising candidate for future optical clocks. For an AOC with lasing frequency resonant with the cavity mode, the linewidth is $1/a^2$ of the Schawlow-Townes linewidth, and the cavity pulling is reduced by a factor of $1/a$, where $a = \Gamma_0/\gamma$ is the ratio between the cavity linewidth Γ_0 and the gain linewidth γ.

However, recent work by Shi et al. shows that the cavity pulling coefficient depends on the cavity detuning [16]. As shown in Fig. 1, the frequency difference between the lasing output and the center of the gain profile (i. e. the cavity pulling) is only linear to the cavity detuning near resonance. When the AOC is far off-resonance, the slope of the cavity pulling curve (hereby referred to as the "differential cavity pulling coefficient") decrease, and even becomes negative when the AOC operates in the anti-resonant cavity regime [16].

Here, we propose a method to experimentally measure the cavity pulling of an AOC by introducing sidebands in the AOC's gain profile. This is achieved by phase-modulating the pumping laser of a four-level AOC based on thermal cesium atoms. By measuring the beat frequency in the lasing output of an AOC pumped by a phase-modulated laser, we can determine the cavity pulling coefficient of the AOC. The advantage of this

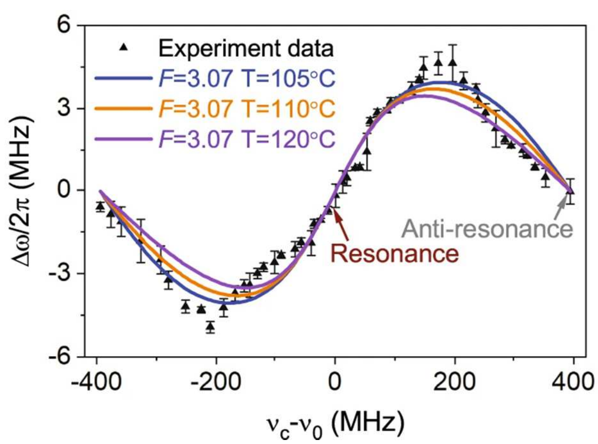

Figure 1. Cavity pulling characteristics of an AOC at different cavity detuning, measured via heterodyne beating (used with permission) [16]. The cavity pulling is linear to the cavity detuning near resonance, then the differential cavity pulling coefficient decreases with increased cavity detuning, eventually becoming negative near anti-response.

method is that, it does not require any additional optical frequency reference, as opposed to using heterodyne beating to measure the frequency shift of an AOC with different cavity detuning.

II. Methods

The proposed experimental setup is shown in Fig. 2 (a), with the pumping and lasing energy levels of Cs shown in Fig. 2(b). We use a 459 nm external cavity diode laser (ECDL) locked to the $6S_{1/2}$-$7P_{1/2}$ pumping transition via modulation transfer spectroscopy (MTS) as the pumping laser for the AOC [17]. The cavity length of the AOC can be fine-tuned using a piezo-electric ceramic (PZT).

We modulate the 459 nm pumping laser with an electro-optical modulator (EOM). This creates sidebands in the pumping laser, with the ±1 order sidebands dominant. Due to Doppler shifts, atoms at rest are pumped by the zeroth order, and atoms with velocities $v_{\pm} = \pm\frac{f_{mod}}{\nu_{pump}}c$ are pumped by the sidebands. Therefore, population inversion is achieved in three separate velocity groups, and sidebands are created in the gain profile of the 4-level AOC. The frequency difference between the adjacent gain peaks is determined by the Doppler shift in thermal atoms [18], and it can be expressed as:

$$\Delta\nu_{gain} = c\left|v_{\pm}\right| = \nu_{AOC}/\nu_{pump} \times f_{mod}. \quad (1)$$

Here, f_{mod} is the modulation frequency of the pumping laser, ν_{AOC} and ν_{pump} are the frequencies of the AOC lasing and pumping transitions, respectively, and $\Delta\nu_{gain}$ is the frequency difference between the zeroth order gain and each sideband.

Since the cavity mode detuning is different for the zeroth order and sidebands of the gain, the lasing output corresponding to the zeroth order and the sidebands experience different cavity pulling. Therefore, assuming $\Delta\nu_{gain}$ is much larger than the linewidth of each separate peak in the gain profile (otherwise they merge together), but much smaller than the cavity linewidth (for the zeroth order and sidebands to have the same differential cavity pulling), the measured beat frequency between the zeroth order and the sidebands of the AOC lasing output becomes $f_{beat} = \left(1 - k_{pulling}\right)\Delta\nu_{gain}$, as illustrated in Fig. 2(c). Here the factor $k_{pulling}$ is the differential cavity pulling coefficient (i.e., the slope of the cavity pulling curve shown in Fig. 1). Using the expression of $\Delta\nu_{gain}$ given by (1), we have:

$$k_{pulling} = 1 - (f_{beat}/f_{mod}) \times \left(\nu_{pump}/\nu_{AOC}\right). \quad (2)$$

Since the frequencies of the lasing and pumping transitions are known quantities, and the modulation frequency f_{mod} can be given with accuracy, we only need to accurately measure the beat frequency in the AOC output to determine the cavity pulling coefficient.

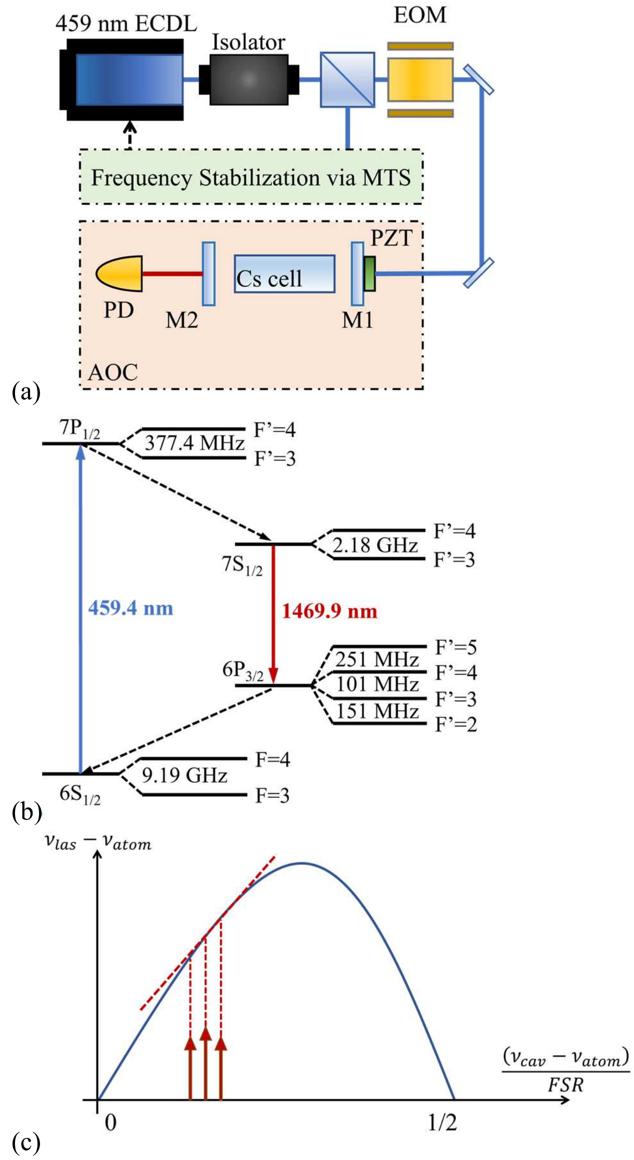

Figure 2. (a) The proposed experimental setup. The pumping laser, which is locked to the Cs 6S1/2-6P1/2 transition frequency stabilization via modulation transfer spectroscopy (MTS) [5], is then phase-modulated by an electro-optical modulator (EOM). The modulated pumping laser then enters the AOC. The beat frequency between the zeroth order AOC output and the sidebands is then detected by the photodetector (PD). (b) Relevant energy levels of cesium. (c) The zeroth order and ±1 sidebands of the gain vs. the cavity-pulling curve. The difference in cavity mode detuning between adjacent gain peaks is equal to detuning between adjacent gain peaks is equal to $\Delta\nu_{gai1}$ given by (1), and the cavity-pulling experienced by the lasing outputs corresponding to adjacent peaks each differs by $k_{pulli1}\ g^{\gamma\Delta\nu}{}_{gai1}$.

III. RESULTS

We use a 459 nm ECDL referenced to a thermal cesium cell via MTS as the pumping laser for the experimental setup, as shown schematically in Fig. 2(a) [17]. The AOC has a cavity length of 170 mm, the cavity mirror on the input side is a plane mirror coated to have maximal transmission for the 459 nm pumping laser and has 50% reflectivity for the 1470 nm clock laser, while the output mirror is plane-concave with a 190 mm radius of curvature and can be selected among mirrors with different reflectivity. The gain medium of the AOC is a 5-cm Cs cell with Brewster-angled windows, heated to 120 ◦C. We measure the lasing power of the AOC while scanning the frequency of the pumping laser, as shown in Fig. 3(a). The maximum AOC lasing power is achieved when the pump laser is on resonance with the $6S_{1/2}$ (F=4) - $7P_{1/2}$ (F'=3) transition. When the laser is tuned to be on resonance with the $6S_{1/2}$ (F=3) - $7P_{1/2}$ transitions, the AOC lasing power is negligible. We also measure the lasing power of the AOC with the pumping laser stabilized to the $6S_{1/2}$ (F=4) - $7P_{1/2}$ (F'=3) transition, and the output mirror reflectivity is set to 80% and 50%, as shown in Fig. 3(b). With MTS-stabilization, the frequency stability of the pumping laser is improved, and pump-induced frequency shifts of the AOC signal are suppressed [19, 20].

IV. DISCUSSION

Beside using Doppler shifts in thermal atoms, we can also use other methods to create multiple separate peaks in the gain profile. For instance, we can use a weak magnetic field to separate the Zeeman sublevels of the lasing transitions, therefore creating multiple separate peaks in the gain profile corresponding to each Zeeman sublevel [21]. By comparing the beat frequency with the calculated Zeeman splitting, we can also determine the cavity pulling coefficient, without utilizing Doppler shifts. Using the measurement of the cavity pulling coefficients, we can also characterize important operational parameters of an AOC, such as the ratio between its gain linewidth and cavity linewidth, which equals to the differential cavity pulling coefficient on resonance. Since the cavity linewidth can be directly calculated from the cavity length and mirror reflectivity, we can obtain the actual linewidth of the AOC's lasing transition from the on-resonance cavity pulling, and analyze broadening effects by comparing with the natural linewidth.

V. CONCLUSIONS

In this abstract, we propose a method of measuring the cavity pulling coefficient of a 4-level AOC based on thermal atoms by phase-modulating the pumping laser. With our proposed method, the slope of the cavity pulling curve can be determined by measuring the self-beat frequency of the AOC lasing output. Compared with heterodyne beating measurements, this method is simpler, as it does not require any additional frequency reference. This method can also be adopted in AOC systems other than those using thermal atoms, like using Zeeman sublevels in cold atoms, for instance. Furthermore, our proposed method for cavity-pulling measurement can serve as a basis for the characterization of AOCs.

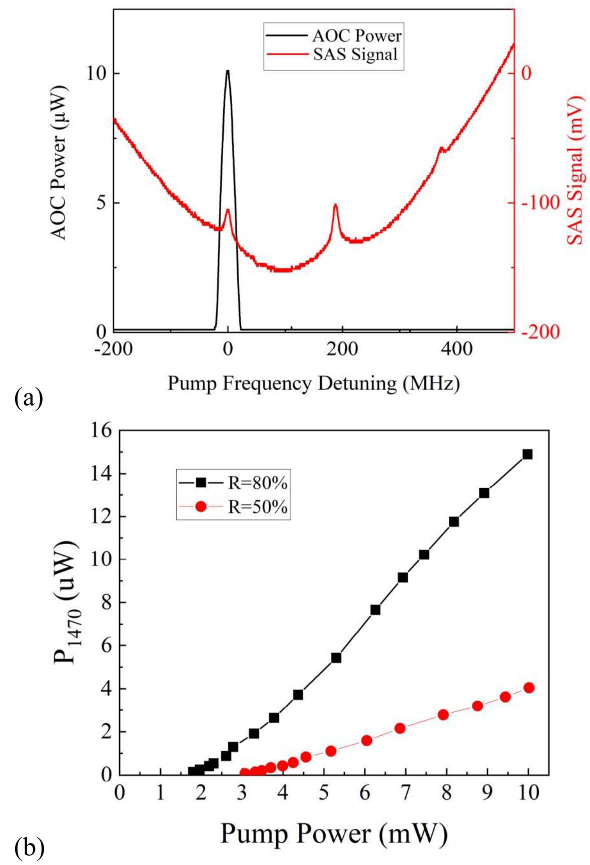

Figure 3. (a) Measured AOC lasing power with varying pump laser frequency detuning from the 6S1/2 (F=4) - 7P1/2 (F'=3) transition, the reflectivity of the output mirror is 80%. The cell temperature is controlled at 120°C, and the pump power is 7.5 mW. (b) Measured AOC lasing power on resonance with varying pump laser power.

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